

charge

$$6.2415 \times 10^{18} \text{ electrons} \equiv 1 \text{ coulomb} \equiv 1 \text{ C}$$

current

ampere \equiv A \equiv coulomb/second

for DC

$$I = \Delta q / \Delta t$$

for variable current

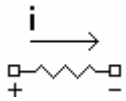
$$i(t) = dq(t) / dt$$

current is the net charge flow through a cross sectional area of a material

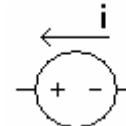
positive flow of current is in the opposite direction of the flow of electrons

labeling two-terminal devices with variables does not assign actual values of + or –

once the + or – is known through analysis, the variable's value will show that



current flows through resistor, passive element from +, higher potential to –, lower potential



current flows through sources from + node, higher potential, around the circuit to – node

sinusoidal waveform

$$(A)\sin(\omega t + \phi) + B$$

A \equiv amplitude, $\omega \equiv$ angular frequency, t \equiv time, $\phi \equiv$ horizontal shift, B \equiv vertical shift

peak value = 2 * Amp, period \equiv T = 1/f = 2 π / ω

potential \equiv voltage \equiv V \equiv J/C

$$V = IR$$

$$v_R(t) = i_R(t)R(t)$$

current flows from higher potential to lower potential, + V to – V

V_{ab} is the voltage drop from point a to b, if $V_{ab} = +$, then point a is at a higher potential than b

symbols on a circuit diagram – +’s, –’s, subscripts are all relative and are used as references

power \equiv P \equiv Watt \equiv W

rate of energy \equiv W \equiv J/s \equiv QV = dW(t)/dt

power absorbed = P = IV

$$P = I_{AB}V_{AB} = I^2R = V^2/R$$

the power delivered by the source is the power dissipated by the rest of the circuit

sum(P_{circuit}) = 0, the sum of the absorbed power by the elements = power delivered

when P of active element is +, the device dissipates power, when P is –, it delivers power

energy \equiv joule \equiv J \equiv work \equiv W

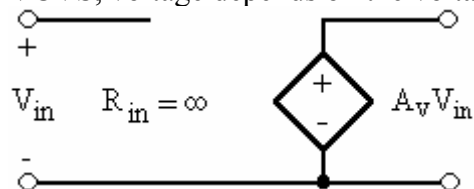
energy absorbed = W = VIT = PT

work = integral(P)dt

ideal voltage source, independent voltage source, constant voltage regardless of circuit, $R_{int} = 0$
 non-ideal is usually mapped as an ideal with a resistor in series
 ideal current source, independent current source, constant current regardless of circuit, $R_{int} = \infty$
 non-ideal is usually mapped as an ideal with a resistance in parallel
 controlled sources allow the possibility of negative resistances

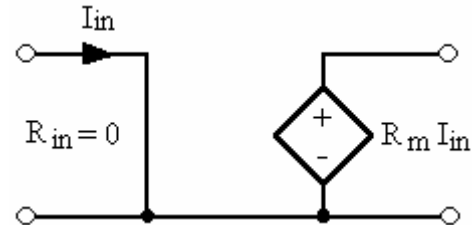
voltage controlled voltage source

VCVS, voltage depends on the voltage, $v(t) = \mu v_x$. $\mu \equiv$ voltage gain



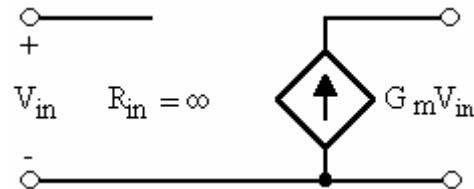
current controlled voltage source

CCVS, voltage depends on the current, $v(t) = r_m v_x$. $r_m \equiv \Omega \equiv$ transfer resistance



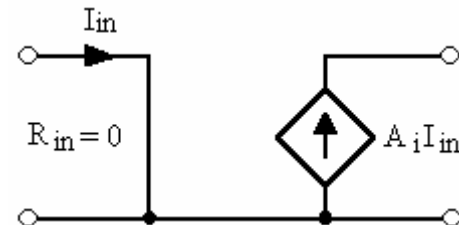
voltage controlled current source

VCCS, current depends on the voltage, $i(t) = g_m v_x$. $g_m \equiv S \equiv$ transfer conductance



current controlled current source

CCCS, current depends on the current, $i(t) = \beta i_x$. $\beta \equiv$ current gain



resistance, ohm $\equiv \Omega$

ohm's law $V_{AB} = I_{AB}R$, v-i characteristic, ideal linear resistor, $v = f(i)$ or $i = f(v)$

resistors always absorb power

when does ohm's law apply

the physical dimensions of the circuit element must be small relative to the wavelength of the sinusoidal frequencies expected to drive the device, called a lumped circuit elements, the opposite of a lumped circuits is a distributed circuit

nodal and loop analysis are needed when there is not a parallel-series circuit

kirchhoff's current law (KCL), nodal analysis

for lumped circuits, the algebraic sum of the currents entering or leaving a node is zero
current sources cannot be in series

make supernodes for floating voltage sources, neither node is connected to the reference

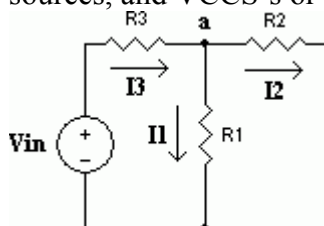
draw a circle around voltage source, label the sources, i.e. V_1 and V_2

when equations are generated, make last equation $V_2 - V_1 = \text{value of voltage source}$

KCL applies for gaussian curves drawn around elements

nodal analysis usually involves less unknowns in larger circuits than loop analysis

nodal analysis is easy when the circuit contains only resistances, interdependent current sources, and VCCS's or $R-I_s-g_m$ networks



$$-I_1 - I_2 + I_3 = 0 = -(V_a - 0)/R_1 - (V_a - 0)/R_2 + (V_{in} - V_a)/R_3$$

kirchhoff's voltage law (KVL), loop analysis

maximum number of equations to n nodes and b branches is $(b - n + 1)$ equations

for lumped circuits, the algebraic sum of the voltages around a closed path is zero

for lumped circuits, the algebraic sum of the voltages around a closed node sequence is zero

the sum of the node voltages in a closed path is zero

the voltage of $V_{AB} = V_{AB} + \dots + V_{YZ}$

$$V_{ab} = V_a - V_b$$

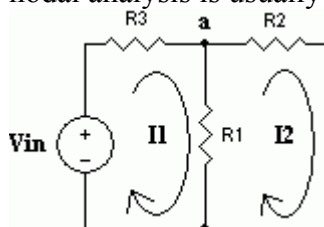
make supermeshes when current sources are common to two more loops

modified loop analysis is easier actually

voltages sources are never connected in parallel

make a reference node 0 V to be ground

nodal analysis is usually better to choose when circuits get bigger



$$+ V_{in} - R_3(I_1) - R_1(I_1 - I_2) = 0$$

$$+ R_1(I_2 - I_1) - I_2(R_3) = 0$$

resistivity $\equiv R = \rho L/A$, Ohm-meters

conductance, siemens $\equiv S$, mho $\equiv \Omega^{-1}$

short circuit \equiv zero resistance, infinite conductance, modeled as ideal wire

open circuit \equiv infinite resistance, zero conductance, modeled as open switch

memoryless device – $v = f(i)$ or $i = f(v)$

memory device – voltage and current relationships satisfy differential equation

in series, for all t

$$i_1 = i_2 = \dots = i_n$$

$$v_{eq} = v_1 + v_2 + \dots + v_n$$

$$R_{eq} = R_1 + R_2 + \dots + R_n$$

$$L_{eq} = L_1 + L_2 + \dots + L_n$$

$$C_{eq} = \frac{1}{C_1^{-1} + C_2^{-1} + \dots + C_n^{-1}}$$

$$v_{R_1} = v_{in} \frac{R_1}{R_1 + R_2 + \dots + R_n}$$

$$v_{C_1} = v_{in} \frac{C_1^{-1}}{C_1^{-1} + C_2^{-1} + \dots + C_n^{-1}}$$

$$i_{L_1} = i_{L_2} = \dots = i_{L_n}$$

in parallel, for all t

$$v_1 = v_2 = \dots = v_n$$

$$i_{eq} = i_1 + i_2 + \dots + i_n$$

$$R_{eq} = \frac{1}{R_1^{-1} + R_2^{-1} + \dots + R_n^{-1}}$$

$$L_{eq} = \frac{1}{L_1^{-1} + L_2^{-1} + \dots + L_n^{-1}}$$

$$C_{eq} = C_1 + C_2 + \dots + C_n$$

$$i_{R_1} = i_{in} \frac{R_1^{-1}}{R_1^{-1} + R_2^{-1} + \dots + R_n^{-1}}$$

$$v_{C_1} = v_{C_2} = \dots = v_{C_n}$$

$$i_{L_1} = i_{L_n} \frac{L_1^{-1}}{L_1^{-1} + L_2^{-1} + \dots + L_n^{-1}}$$

nonideal sources can be modeled as an ideal independent voltage source (with zero resistance) in series with a resistor or an ideal independent current source (with infinite resistance) in parallel with a resistor

to measure the voltage of a battery, supply a typical current flow through the battery so that the internal resistance will be accounted for. internal resistance is not seen by very small current supplied by a voltmeter

modified loop analysis

add auxiliary voltage variables for current sources common to two or more loops

apply kvl

write constraint equations determined by the dependent current sources

solve in matrix form

modified nodal analysis

the unknown are the usual nodal voltages plus auxiliary currents

auxiliary currents:

- current through independent voltage sources
- current through dependent voltage sources
- current through short-circuit elements (0 V independent)
- controlling currents of dependent sources
- currents declared as output quantities

for every element x whose current source has been chosen as an auxiliary current:

temporarily replace that element by an independent current source having the value I_x

write an equation that relates the elements of the original to the new setup

solve in matrix form

writing loop equations for non-planar circuits

keep track of the wires, when a wire hops over another, your loop equations should too

a/d converter

converts signals to a discrete quantization

d/a converter

converts a binary number

$[b_{n-1}, \dots, b_2, b_1, b_0]$, where b_i is either 1 or 0

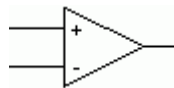
to an output voltage

$v_{out} = [b_{n-1}2^{n-1} + \dots + b_2n^2 + b_1n^1 + b_0n^0]E_0$, E_0 is some constant

the circuit that allows the voltage to be converted is a R-2R ladder network

voltage division is applied and switches are used to direct current through different branches

ideal op amp



ideal op-amps supply voltage without loading the circuit or,

an ideal op amp supplies voltage to a resistance without letting the resistance load the circuit

voltage gain $\equiv A = V_{out} / V_{in}$

$v_d = v_+ - v_-$

usually modeled with a VCVS, R_{out} being a wire, and R_{in} being open

does not load the circuit

$$R_{in} = \infty \quad R_{out} = 0$$

the full output voltage appears across any circuit connected to the output part

$$v_+ = v_- \quad A = \infty$$

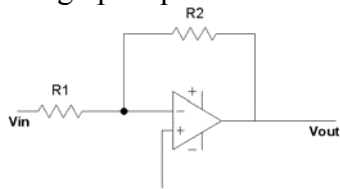
no current runs through op-amp

$$i_+ = 0 \quad i_- = 0$$

nonideal op amp

$$V_{out} = AV_d$$

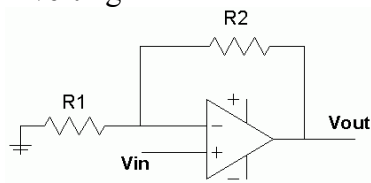
inverting op amp



$$, V_{out} = -\frac{R_2}{R_1} V_{in}$$

input and output voltages are always positive

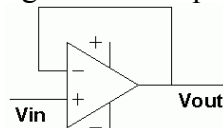
noninverting



$$, V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$

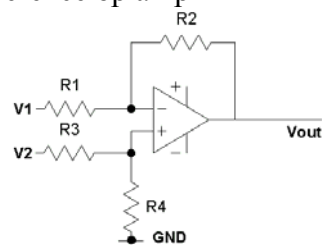
does not reverse polarity

voltage follower op amp



used to prevent the load, R_L , from drawing current directly from the source by v_{in}
it supplies current directly to the load

difference op amp



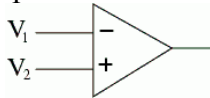
used to compare voltages, or to amplify the difference
magnitude scaling, scale the ratio of the resistance and achieve the same gain
useful to make the resistance values more practical

saturation and the active region of the op amp

when input increases or decreases beyond a certain range, the output will clamp at minimum or maximum voltage levels, in between that is the active region.

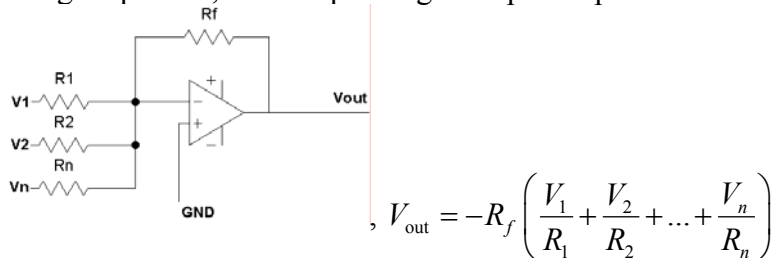
the critical threshold voltages of v_d , difference between the terminals, where saturation occurs is $\pm V_{sat}/A$.

comparator



used to compare input voltage with reference voltage, it utilizes the saturation levels

summing amplifiers, multi-input single-output amplifiers



general scale of multi-input single-output

$$V_{out} = -(a_1V_{a1} + a_2V_{a2} + \dots + a_nV_{an}) + (b_1V_{b1} + b_2V_{b2} + \dots + b_mV_{bm})$$

feedback resistor is generalize to 1Ω , then the resistances would be scaled to more practical values without changing the gain

prototype design

set $G_f = 1 \text{ S}$

$$G_{ai} = a_i \text{ S for } i = 1, \dots, n$$

$$G_{bi} = b_i \text{ S for } i = 1, \dots, m$$

total conductance incident on the inverting terminal will need to be the same as the total conductance incident on the noninverting terminal by a proper choice of ΔG and G_{ground}

compute G_{ground} and ΔG so the total conductance on the two terminals are equal

$$\text{define } \delta = (1 + a_1 + \dots + a_n) - (b_1 + \dots + b_m)$$

if $\delta > 0$, then set $G_{ground} = \delta$ and $\Delta G = 0$

if $\delta \leq 0$, then set $G_{ground} = \text{some value so that } G_{ground} = 1 \text{ S and } \Delta G = |\delta| + G_{ground}$

scaling to achieve practical element values

multiply all the conductance incidences at the inverting and non-inverting terminals by a constant K_a and K_b respectively, good if $K_a = K_b$

linearity theorem, for linear circuits

output voltage or current is linearly related to all of the independent sources by the equation:

$$y = a_1u_1 + a_2u_2 + \dots + a_mu_m$$

where y is output, a 's are dimensionless quantities and u 's are the independent sources

superposition, for linear circuits

finding the contribution from one independent source by setting all other to zero
this is like setting up a matrix table

if there are fixed sources, then all of them are represented by one constant, the fixed sources contribute a fixed amount to the circuit, the constant

$$v_{out} = Ai_s + B$$

superposition does not apply to power calculations
when setting values to zero, treat voltage sources as wires and current sources as open
leave dependent sources alone and calculate the contribution in terms of those values

proportionality property

when any independent sources are acting alone, the output is proportional to that single input
if a linear circuit is dependent only upon one sources, then when that source is multiplied by a constant, the current and voltage of the circuit will also be multiplied by that constant

source transformation

when a voltage source in series with a resistor is converted to a current in parallel with that same resistor value, or the other way around, this is the equivalent two-terminal network, it applies to both dependent and independent voltage and current sources

when doing source transformation on dependent sources, it is important to keep the controlling source unaffected, not unchanged, while transforming the circuit

make little square circuits when the sources are deep in the network

modified superposition analysis

when there are dependent sources, the analysis contains more variables, this modified analysis shows how to deal with this

replace the dependent source with an independent source and label it
use superposition to solve for the unknown in terms of the different sources alone
also solve for the controlling variables of the replaced dependent source
restore the constraint replacement

this method can also be used to replace R with a dependent voltage source R_i , and G with a dependent current source G_v where the original circuit current's direction is kept the same

then you will replace and restore the dependent source that you created

bilinear form theorem

$$\frac{\text{output}}{\text{input}} = \frac{b_0 + b_1 x}{a_0 + a_1 x}, \text{ i.e. } \begin{bmatrix} 1 & R & A \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ a_0 \end{bmatrix} = R A a_1, \text{ where } A = V_{\text{out}} / V_{\text{in}}$$

x is a resistance or ant dependent source parameter

this ratio can be ($V_{\text{out}} / V_{\text{in}}$, $V_{\text{out}} / I_{\text{in}}$, $I_{\text{out}} / I_{\text{in}}$, $I_{\text{out}} / V_{\text{in}}$, R_{in} or G_{in})

for the relationship between some varying resistance R and gain A , the matrix above

thevenin theorem for passive networks, $v_{\text{oc}} = R_{\text{th}} i_{\text{sc}}$

for a linear circuit containing resistances and independent sources, there exists an equivalent two-terminal network consisting of an equivalent thevenin resistance R_{th} , when all the independent voltage sources are deactivated, in series with an open-circuit voltage v_{oc} , when no other network is attached

find R_{th} by deactivating all independent sources

voltage sources become short and current sources become open

if the circuit is a series-parallel, use inspection to reduce the circuit

if it is not a series-parallel, use KCL or KVL, whatever makes finding the unknown easier

norton theorem for passive networks, $v_{\text{oc}} = R_{\text{th}} i_{\text{sc}}$

for a linear circuit containing resistances and independent sources, there exists an equivalent two-terminal network consisting of an equivalent thevenin resistance R_{th} , when all the independent voltage sources are deactivated in parallel with an independent short circuit current source i_{sc}

proceed to reduce the circuit like the thevenin theorem procedures

when a network contains no independent sources, $v_{\text{oc}} = i_{\text{sc}} = 0$ with a single resistance R_{th}

to find v_{oc} , use thevenin or norton equivalent

to find i_{sc} , connect a wire across terminals

thevenin and norton equivalents for active networks, containing dependent sources and op amps

if it also has any independent sources, deactivate them to find the network equivalent, R_{th}

apply a voltage or current source to the two terminals to find R_{th}

use $R_{\text{th}} = v_s / i_s$ to find either v_s or i_s

use v_s or i_s to find the unknown part of the dependent source

remember that R_{th} is the equivalent resistance depending on what two terminals you pick and depending on what state the circuit is at that instant, the state will especially change when dealing with switches

when you replace a circuit with a thevenin, you have $v_{AB} = R_{th}i_s + v_{oc}$, where $R_{th}i_s$ is the voltage created by a fictitious current source, and v_{oc} is the voltage created by the independent sources in the circuit across the two terminals. these two things will allow you to find v_{AB} , the total voltage across the two terminals opening and closing

when you replace a circuit with a norton, you have $i_A = v_s / R_{th} - i_{sc}$, where v_s / R_{th} is the current created by a fictitious voltage source, and i_{sc} is the current created by the independent sources in the circuit across the two terminals. these two things will allow you to find i_A , the total current across the two terminals

thevenin and norton equivalents from measured data

you can make two measurements in the laboratory of v_{ab} to make two equations and two unknowns, multiple thevenin equivalents can be made to model nonlinear circuits

other theories

any circuit that has a horizontal v-i characteristic in the v-i plane has a Norton equivalent but not a Thevenin

any circuit that has a vertical v-i characteristic in the v-i plane has a Thevenin equivalent but not a Norton

norton and thevenin equivalents can not be made with v-i characteristics described as points

(there is only one voltage and current for all voltages appearing across the terminals) or as planes (any i is possible for each v)

an independent current source does not have a thevenin equivalent

an independent voltage source does not have a norton equivalent

to find out whether there is equivalent, if the determinant of the matrix of vectors of unknown voltages and currents containing, v_1 , equaling vectors of independent source values containing i_1 or zero values is not equal to zero, then the network has a thevenin equivalent

$$\begin{bmatrix} M \\ \dots \end{bmatrix} \begin{bmatrix} v_1 \\ \dots \end{bmatrix} = \begin{bmatrix} i_1 \\ \dots \end{bmatrix}$$

or, if the determinant of the coefficients matrix in the modified loop equation is not equal to zero, then there is a Norton

if R_{th} is not equal to 0, then there is a norton equivalent

if $1 / R_{th}$ is not equal to zero, there is a thevenin

maximum power

maximum power is transferred to the load when $R_L = R_{th}$
 this is important, i.e., when trying to match loud speaker “resistances” to output “resistances” of a stereo amplifier or when trying to get as much power as possible out of an antenna and into a receiver, but it is not important for power transmission networks

the condition where maximum power transfer is delivered to a load network will occur when
 $v = 0.5v_{oc}$

where v is the terminal voltage, the i through v will thus be

$i = 0.5v_{oc} / R_{th}$
 and the $p_{L, max}$ will be $v_{oc}^2 / 4R_{th}$

dual circuits

two circuits are dual if KCL, KVL, and v - i branch equations in one circuit become KVL, KCL, and v - i branch equations upon interchanging $i_k \leftrightarrow v_k^*$ and $v_k \leftrightarrow i_k^*$. any solution in N leads to a solution to N^* and relationship among variables in N lead to relationship among variables in N^* . N^* will only exist if N is planar. N and N^* must have the same number of branches, but not nodes. a dual circuit elements guide will help build dual circuits.

there are problems making dual circuits when:

the circuit is not series-parallel

the circuit contains controlled sources

the reference direction of the circuit elements play an important role in the solution

when this happens: follow procedures below

procedures for determining the connection of dual network branches

draw the given planar circuit N without branch crossings, and identify the meshes (regions)

place a node of N^* inside each mesh and one in the outside, infinite region

for each branch b in N that is on the boundary of two regions, draw the dual branch b^* joining the nodes placed in each of these regions. if the branch arrow in N is clockwise around the node of N^* , the arrow for the corresponding branch in N^* is away from the node of N^*

inductance, a coiled conductor

measures the magnitude of the voltage induced by a change in the current through an inductor

energy storage occurs, the induced voltage is proportional to the derivative of the current, the inductance of the coil which is a constant of proportionality is L

$$i_L(t) = i_L(t_0)e^{\frac{-R_{th}}{L}(t-t_0)} \quad v_L(t) = L \frac{di_L(t)}{dt} \quad i_L(t) = \frac{1}{L} \int_{-\infty}^{t_0} v_L(\tau) d\tau \quad p_L(t) = v_L(t)i_L(t)$$

energy stored in a inductor

$$W_L(t_0, t_1) = L \int_{i_L(t_0)}^{i_L(t_1)} i_L di_L = \frac{1}{2} Li_L^2(t_1) - \frac{1}{2} Li_L^2(t_0), \quad W_L(t) = \frac{1}{2} Li_L^2(t)$$

stored energy is independent of voltage waveform in between t_i and t_f , only initial and final if the voltage across an inductor is bounded, the current through the inductor is continuous

the inductor is a lossless device, all energy stored is returned

inductors initially behave like an open in the circuit

inductors behave like a short or a zero resistance wire after a very long time

capacitor, separated conductors

current is proportionality to the time rate of change of its voltage, the ability to produce a current from the changes in the voltages across it

$$q(t) = Cv_C(t)$$

$C_1v_1(t) + C_2v_2(t) + \dots = 0$, junction of more than one capacitors

$$v_C(t) = v_C(t_0)e^{\frac{-1}{R_{th}C}(t-t_0)} \quad i_C(t) = C \frac{dv_C(t)}{dt} \quad v_C(t) = \frac{1}{C} \int_{-\infty}^t i_C(\tau) d\tau \quad p_C(t) = v_C(t)i_C(t)$$

energy stored in a capacitor

$$W_C(t_0, t_1) = C \int_{t_0}^{t_1} p_C(\tau) d\tau = \frac{1}{2} Cv_C^2(t_1) - \frac{1}{2} Cv_C^2(t_0), \quad W_C(t) = \frac{1}{2} Cv_C^2(t)$$

stored energy is independent of voltage waveform in between t_i and t_f , only initial and final

if the current through a capacitor is bounded, the voltage across the capacitor is continuous, unless two are connected in parallel or when some capacitors and some voltage sources form a loop, but it really doesn't break the rule, it's just fast

the capacitor is a lossless device, all energy stored is returned

initially capacitors behave like a short or a zero resistance wire

after a long time capacitors behave like an open in the circuit

first-order RL and RC circuits

circuits have behaviors modeled by differential equations
 the solution to a first-order differential equation is a waveform, or a signal or a response
 the solution works for all continuous and piecewise continuous time functions
 a solution to a differential equation means that the waveform satisfies the given differential equation with the proper initial condition

source free or zero input response

$$\frac{di_L(t)}{dt} = -\frac{R}{L}i_L(t) \quad i_L(t) = i_L(t_0)e^{-\frac{R}{L}(t-t_0)}$$

$$\frac{dv_C(t)}{dt} = -\frac{1}{RC}v_C(t) \quad v_C(t) = v_C(t_0)e^{-\frac{1}{R_{th}C}(t-t_0)}$$

$\tau = L/R = RC$, time it takes the source-free circuit response to drop to e^{-1} of the initial value

dc or step response of first-order circuits

for a linear resistive circuit with constant sources

$$v_L(t) = V_{oc} - R_{th}i_L(t) \quad i_C(t) = \frac{V_{oc} - v_C(t)}{R_{th}}$$

$$\frac{di_L(t)}{dt} = -\frac{R_{th}}{L}i_L(t) + \frac{R_{th}}{L}I_{sc} \quad \frac{di_L(t)}{dt} = -\frac{R_{th}}{L}i_L(t) + \frac{1}{L}V_{oc}$$

$$\frac{dv_C(t)}{dt} = -\frac{1}{R_{th}C}v_C(t) + \frac{1}{R_{th}C}V_{oc} \quad \frac{dv_C(t)}{dt} = -\frac{1}{R_{th}C}v_C(t) + \frac{1}{C}I_{sc}$$

V_{oc} implies in series with R_{th} I_{sc} implies parallel with R_{th}

general solution to the equations above, $x(t) = F\tau + [x(t_0^+) - F\tau]e^{-\frac{(t-t_0)}{\tau}}$

$$x(t) = i_L \quad F\tau = V_{oc} / R_{th} = I_{sc}$$

$$x(t) = v_C \quad F\tau = V_{oc}$$

when $[x(t) = \text{constant}]$ satisfies a differential equation, it is called the equilibrium state

$F\tau$ is the equilibrium state

when $\tau > 0$, or when $R_{th} > 0$, $C > 0$, and $L > 0$, or when the circuit is passive

$$i_L(t) = i_L(\infty) + [i_L(t_0^+) - i_L(\infty)]e^{-\frac{R_{th}}{L}(t-t_0)}$$

$$v_C(t) = v_C(\infty) + [v_C(t_0^+) - v_C(\infty)]e^{-\frac{1}{R_{th}C}(t-t_0)}$$

these equations are easier to use if you can use them because only the initial and final values of i_L and v_C are needed

when looking at a first-order RC or RL circuit

does have dependent sources?

does it not have independent sources?

if it also has any dependent sources, deactivate them to find the network equivalent

apply a voltage or current source to the two terminals from which you want to find the equivalent resistance

use $R_{th} = v_s / i_s$ to find either v_s or i_{sc}

use v_s or i_{sc} to find the unknown part of the dependent source or whatever else

now that you have R_{th}

does it have the initial inductor current or capacitor voltage given?

is it not driven by a dependent source, AT the specific time?

use $i_L(t) = i_L(t_0)e^{-\frac{R_{th}}{L}(t-t_0)}$ or $v_C(t) = v_C(t_0)e^{-\frac{1}{R_{th}C}(t-t_0)}$

is it driven by a dependent source, AT the specific time?

use $i_L(t) = i_L(\infty) + [i_L(t_0^+) - i_L(\infty)]e^{-\frac{R_{th}}{L}(t-t_0)}$ or $v_C(t) = v_C(\infty) + [v_C(t_0^+) - v_C(\infty)]e^{-\frac{1}{R_{th}C}(t-t_0)}$

LC circuits

$$v_C(t) = K \cos(\omega t + \theta) \quad v_C(t) = V_0 \cos\left(\frac{1}{\sqrt{LC}}t\right)$$

$$i_L(t) = K \cos(\omega t + \theta) \quad i_L(t) = V_0 \sqrt{\frac{C}{L}} \sin\left(\frac{t}{\sqrt{LC}}\right)$$

source-free LC circuits are undamped

first-order RC op amp circuits

ideal differentiator op amp

$$v_{out}(t) = -RC \frac{dv_{in}(t)}{dt}$$

ideal integrator op amp

$$v_{out}(t) = -\frac{1}{RC} \int_0^t v_{in}(\tau) d\tau$$

superposition and linearity

again, like before, one sets all the independent sources to zero and computes the response due to each initial condition with all other initial conditions set to zero. the sum of all of the responses to each of the independent sources plus the individual initial condition responses yields the complete circuit response, by the principle of superposition

$$v_C(t) = a_1 \left[\frac{1}{C} \int_{-\infty}^t i_{C1}(\tau) d\tau \right] + a_2 \left[\frac{1}{C} \int_{-\infty}^t i_{C1}(\tau) d\tau \right] = a_1 v_{C1}(t) + a_2 v_{C2}(t)$$

for example, if there two independent sources and an initial inductor current, set the sources to zero, find the response due to the initial condition of initial current. set the initial current and one of the sources to zero, and find the contribution from the other source, then find do the same thing for the other source

superposition is good when you want to change one of the sources or just the initial condition because you do not have to recalculate the equivalent circuit network

voltage and current in these circuits may not always behave continuously, this may be the case when capacitors, inductors, and switches turn into shorts and opens

when $v_{in}(t) = -18u(-t) + 9u(t)$, $v = -18$ V when $t < 0$ and $v = 9$ V when $t > 0$

the general form of an RC and RL circuit has the form

$$x(t) = X_e + [x(t_0^+) - X_e]e^{-\frac{(t-t_0)}{\tau}}, \text{ where } X \text{ is the voltage or current level, and where}$$

$$t_2 - t_1 = \tau \ln \left[\frac{X_1 - x(\infty)}{X_2 - x(\infty)} \right], \text{ is the elapsed time formula, the subscripts match up, } X \text{ and } t$$

series RLC

$$v_C'' + \frac{R}{L}v_C' + \frac{1}{LC}v_C = F$$

parallel RLC

$$i_L'' + \frac{1}{RC}i_L' + \frac{1}{LC}i_L = F$$

to find initial conditions, replace each capacitor by an independent current source with value $v_C(0^+)$ and each inductor with an independent current source with value $i_L(0^+)$

solutions to RLC,

$$s_1, s_2 = \frac{-b \pm \sqrt{b^2 - 4c}}{2} \quad \sigma = \frac{b}{2} \quad \omega_d = \frac{\sqrt{b^2 - 4c}}{2}$$

real and distinct, overdamped

$$x(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} + X_F \quad X_F = \text{final value of either voltage or current}$$

$$x(0^+) = K_1 + K_2 + X_F \quad x'(0^+) = s_1 K_1 + s_2 K_2$$

complex and distinct, underdamped

$$x(t) = e^{-\sigma t} [K_1 \cos(\omega_d t) + K_2 \sin(\omega_d t)] + X_F$$

$$x(0^+) = K_1 + X_F \quad x'(0^+) = -\sigma K_1 + \omega_d K_2$$

$$s_1 = -\sigma + j\omega_d \quad s_2 = -\sigma - j\omega_d$$

real and equal, critically damped

$$x(t) = (K_1 + K_2 t)e^{st} + X_F$$

$$x(0^+) = K_1 + X_F \quad x'(0^+) = sK_1 + K_2$$

when dealing with the initial conditions of an RLC circuit, you can turn capacitors and inductors into independent voltage and current sources

complex number review

$$\begin{aligned}
 a + jb &= \rho \angle \theta = \rho e^{j\theta} & \rho &= \sqrt{a^2 + b^2} & \tan(\theta) &= b/a \\
 (a + jb)(c + jd) &= \text{foil} = \rho_1 \rho_2 \angle (\theta_1 + \theta_2) & & & \frac{(a + jb)}{(c + jd)} &= \text{rationalize} = \frac{\rho_1}{\rho_2} \angle (\theta_1 - \theta_2) \\
 \text{Re}[\rho e^{j\theta}] &= \rho \cos(\theta) & -1/j &= j & \sin(\theta) &= \cos(\theta - 90^\circ) \\
 \mathbf{V} = V_m \angle \phi &= V_m \cos(\omega t + \phi) & |a + jb| &= \sqrt{a^2 + b^2} & \angle(a + bj) &= \tan^{-1}(b/a), \text{ watch quadrant}
 \end{aligned}$$

impedance

$$Z_R = R \quad Z_C = \frac{1}{j\omega C} \quad Z_L = j\omega L \quad R = \text{Re}[Z] \quad \text{Re}[\mathbf{V}/Z] = i(t) \quad \text{Re}[IZ] = v(t)$$

impedance magnitude is max, or admittance min, and values are purely real when $\text{Im}[Z] = 0$

when $\text{Im}[Z] > 0$, you have an inductor

capacitor voltage and inductor current are always continuous

$$p(t) = v(t)i(t) \quad P_{av} = \frac{1}{T} \int_0^T v(t)i(t) dt$$

$$P_{av,R} = \frac{V_m^2}{2R}, \text{ for SSS} \quad P_{av,R} = V_{eff}^2 R, \text{ for periodic waveforms}$$

$P_{av} = V_{eff} I_{eff} \cos(\theta_v - \theta_i)$ ($\theta_v - \theta_i$) = the angle of the impedance of the two terminal element
effective value = rms

$$\text{sinusoidal } F_{eff} = \frac{F_m}{\sqrt{2}} \quad \text{triangular } F_{eff} = \frac{F_m}{\sqrt{3}} \quad \text{square } F_{eff} = F_m \quad \mathbf{S}_{\text{source}} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3$$

complex power absorbed = $\mathbf{S} = \mathbf{V}_{\text{eff}} \mathbf{I}_{\text{eff}}^*$ $\mathbf{I}_{\text{eff}}^*$ = the complex conjugate

average power - $\text{Re}[\mathbf{S}] = P$

reactive power - $\text{Im}[\mathbf{S}] = jQ$

apparent power - $|\mathbf{S}| = \sqrt{P^2 + Q^2}$

$$\text{power factor} = \text{pf} = \frac{P_{av}}{|\mathbf{S}|} = \cos(\theta_v - \theta_i) \quad \text{pf} = \frac{P}{\sqrt{P^2 + Q^2}}$$

pf lagging $0 < (\theta_v - \theta_i) < 180$ pf leading $0 < (\theta_i - \theta_v) < 180$

$Q > 0$ pf lagging $Q < 0$ pf leading

low pf, large Q, large loss

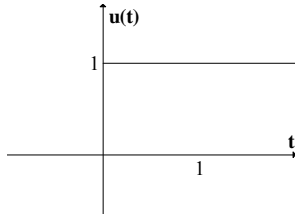
high pf, low Q, small loss

there is maximum power transfer to a load when

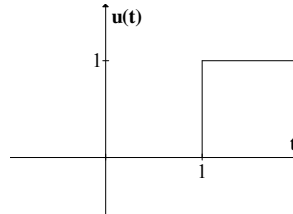
$$Z_L = Z_S^* \quad R_L = R_S \quad X_L = -X_S \quad P_{\text{max,av}} = \frac{V_{S,eff}^2}{4R_S}$$

step function

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

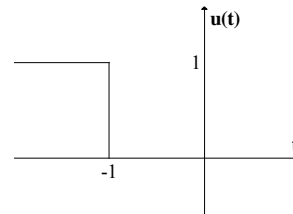


$$u(t-T) = \begin{cases} 0, & t < T \\ 1, & t \geq T \end{cases}$$



$$u(t-T), T=1$$

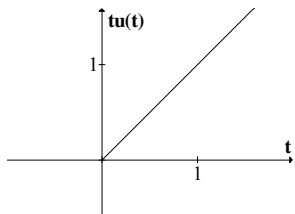
$$u(T-t) = \begin{cases} 0, & t > T \\ 1, & t \leq T \end{cases}$$



$$u(T-t), T=1$$

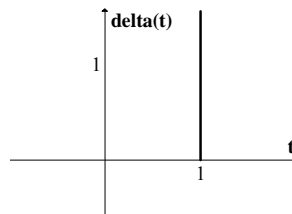
ramp

$$r(t) = tu(t)$$



delta function

$$\delta(t) = \frac{du(t)}{dx} \quad \int_{0^-}^{\infty} \delta(t) dt = 1 = u(0^+) - u(0^-)$$



$$\delta(t-1)$$

sifting property

$$\int_{-\infty}^{\infty} f(\tau) \delta(\tau - t_i) d\tau = f(t_i)$$

laplace transform

$$\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t) e^{-st} dt$$

$$\mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi j} \int_{\Gamma} F(s) e^{st} ds$$

$$\mathcal{L}[tf(t)u(t)] = -\frac{dF(s)}{ds}$$

$$\mathcal{L}[e^{-at} f(t)] = F(s+a)$$

$$\mathcal{L}[f(t-T)u(t-T)] = e^{-sT} F(s)$$

$$\mathcal{L}[v_c(t)] = \mathcal{L}\left[\frac{1}{C} \int_{-\infty}^t (i_c)(\tau) d\tau\right] = V_c(s) = \frac{1}{sC} I_c(s) + \frac{1}{s} v_c(0^-)$$

$$\mathcal{L}[i_c(t)] = I_c(s) = CsV_c(s) - Cv_c(0^-)$$

$$\mathcal{L}[v_L(t)] = V_L(s) = LsI_L(s) - Li_L(0^-)$$

$$\mathcal{L}[Z_C(t)] = Z_C(s) = \frac{1}{Cs}$$

$$\mathcal{L}[Z_L(t)] = Z_L(s) = Ls$$

$$\mathcal{L}[i_L(t)] = \mathcal{L}\left[\frac{1}{L} \int_{-\infty}^t (v_L)(\tau) d\tau\right] = I_L(s) = \frac{1}{Ls} V_L(s) + \frac{1}{s} i_L(0^-)$$

(if you have parallel RLC, C and L voltages are equal and can be exchanged in above)
 (added v go in series, added i go in parallel)

laplace transform technique

divide throughout to get coefficient of highest denominator term to 1

$$\mathcal{L}^{-1}\left[\frac{5}{3s+4}\right] = \mathcal{L}^{-1}\left[\frac{(5/3)}{\cancel{(3/3)}s+(4/3)}\right] = \mathcal{L}^{-1}\left[\frac{5/3}{s+\sqrt{4/3}^2}\right]$$

to get in correct form for sine, for example

$$\mathcal{L}^{-1}\left[\frac{\frac{5/3}{\sqrt{4/3}}\sqrt{4/3}}{s+\sqrt{4/3}^2}\right] = \frac{5/3}{\sqrt{4/3}}\mathcal{L}^{-1}\left[\frac{\sqrt{4/3}}{s+\sqrt{4/3}^2}\right] = \frac{5/3}{\sqrt{4/3}}\mathcal{L}^{-1}[\sin\sqrt{4/3}t]$$

time shift technique

$F(s) = e^{t-3} \sin(t-1)u(t-1)$, there is a difference in paramters for different parts of $F(s)$

$$e^{t-3} = e^{t-3+1-1} = e^{t-1}e^{t-3+1} = e^{(t-1)}e^{-2}$$

$$F(s) = e^{(t-1)}e^{-2} \sin(t-1)u(t-1)$$

$$-t(u(t)+u(t-2)) = -tu(t) - (t-2+2)u(t-2) = -tu(t) - (t-2)u(t-2) - 2u(t-2)$$

partial fractions technique

$$F(s) = \frac{s+1}{(s+1)(s+2)^2(s^2+3)} = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2} + \frac{Ds+E}{(s^2+3)}$$

$$A = (s+1)F(s)\big|_{s=-1} \quad B = (s+2)^2 F'(s)\big|_{s=-2} \quad C = (s+2)^2 F(s)\big|_{s=-2} \quad \text{after plugging in A,B,C}$$

$$E \Rightarrow F(s)\big|_{s=0} = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2} + \frac{\cancel{D(0)}+E}{(s^2+3)}, \quad D \text{ cancels}$$

$$D \Rightarrow F(s)\big|_{s=1} = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2} + \frac{D(1)+E}{(s^2+3)}, \quad \text{using value of E from before}$$

cramer's rule

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} C \\ D \end{bmatrix} \quad A = \frac{\det \begin{bmatrix} C & b \\ D & d \end{bmatrix}}{\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}} = \frac{dC - bD}{ad - bc} \quad B = \frac{\det \begin{bmatrix} a & C \\ c & D \end{bmatrix}}{\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}} = \frac{aD - cC}{ad - bc}$$

for two capacitors in parallel

$$v_{C_1}(0^+) = v_{C_2}(0^+) = \frac{C_1 v_{C_1}(0^-) + C_2 v_{C_2}(0^-)}{C_1 + C_2}$$

RC op-amp modeled by two cacitors

$RC_i = 2T$, to find new C_i that hugs the ramp of the original RC circuit output

$$\text{using } Q = CV, \quad V_{out} = V_{in} C / C_f$$

initial charge and current on capacitor and inductor

attached positive voltage source in series with capacitor, negative to positive

attached current source in parallel with inductor, current runs from positive to negative

transfer function

$$H(s) = \frac{n(s)}{d(s)} = K \frac{(s - z_1)(s - z_2) \cdots (s - z_m)}{(s - p_1)(s - p_2) \cdots (s - p_n)}, \quad s = z = \text{zero}, \quad s = p = \text{pole}, \quad \text{in units } s^{-1}$$

the circuit corresponding to this transfer function is stable if a bounded linear input produces a bounded output, or if the poles lie in the open left hand plain

zero-input response – inputs set to zero

zero-state response – initial conditions are zero

complete response – zero-input + zero-state response

transient response – non-periodic part of the response, stuff that goes to 0 when t goes to infinity

steady-state response – periodic part of the response, stuff that remains as t goes to infinity

natural response – part of the complete response with same exponents as the zero-input response

in the form $e^{-\sigma t} (A \cos(\omega t) + B \sin(\omega t))$, σ and ω come from poles

find A and B by looking at form of transform function, either sine or cosine, so one will go to zero, and and by looking at the initial conditions at $t = 0$

forced response – part of the complete response with same exponents as the input excitation,

provided that the input excitation has exponents different from those of the zero-input response

complete response can be decomposed into natural and forced responses if the input excitation is: dc, real exponential, sinusoidal, exponentially modulated or damped sinusoidal

if all the inductors and capacitors of the transfer function are seen in the equation,

$d(n)$ degree = n, and if the input is in the form,

$$B \cos(\omega t + \phi),$$

then the magnitude of the response is,

$$B = A |H(j\omega)|,$$

and the phase shift is,

$$\phi = \theta + \angle H(j\omega_{\text{fixed}})$$

magnitude (frequency) response

$$|H(j\omega)| \text{ for finite non-negative } \omega$$

$$|(j\omega)^2 + 2j\omega + 5| = |-\omega^2 + 2j\omega + 5| = \sqrt{(5 - \omega^2)^2 + (2\omega)^2}, \text{ ignore } j$$

phase (frequency) response

$$\angle H(j\omega) \text{ for finite non-negative } \omega$$

impulse response

$$H(s) \mathcal{L}[\delta(t)] = H(s)$$

step response

$$H(s)\mathcal{L}[u(t)] = H(s)/s$$

initial-value theorem

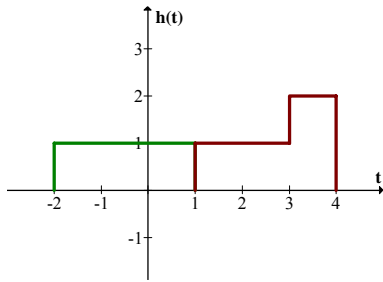
$$\lim_{s \rightarrow \infty} sF(s) = f(0^+), \text{ for strictly proper rational functions of } s$$

final-value theorem

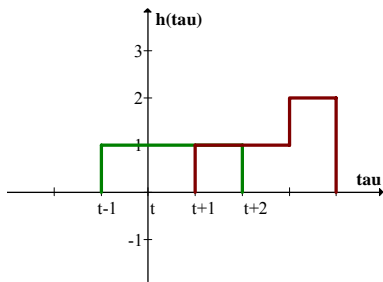
$$\lim_{s \rightarrow 0} sF(s) = f(\infty), \text{ if the poles of } F(s) \text{ lie only in the open left half complex plane, and optionally } \textit{one} \text{ non-repeating pole at the origin}$$

convolution

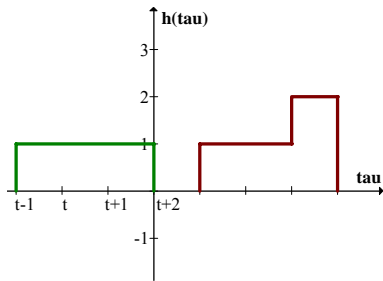
$$y(t) = f(t) * h(t) = h(t) * f(t) = \int_{-\infty}^{\infty} h(t-\tau)f(\tau)d\tau$$



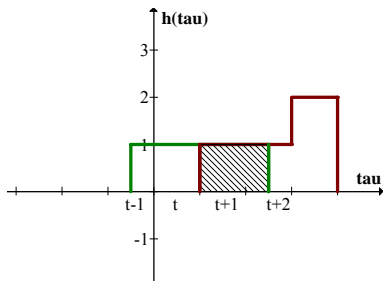
pick a function to switch into $t - \tau$



flip that function into $t - \tau$
label the endpoints of the shifted function

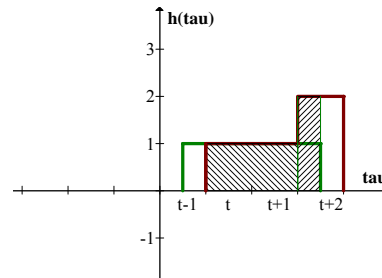


shift that function left enough to clear the start of the other function and start shifting right

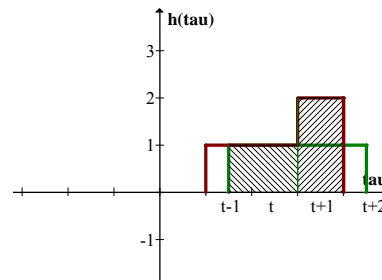


for $-1 < t < 1$ |

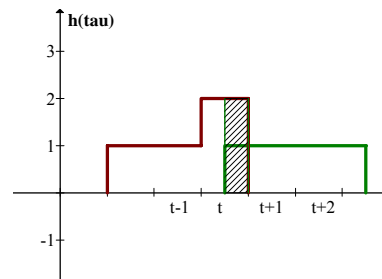
when the function was flipped, it was at $t = 0$, shifting it -1 put it at the start of 2nd function



for $1 < t < 2$ |



for $2 < t < 4$ |



for $4 < t < 5$ |